ADDING FRACTIONS

SETUP the EL-W531 EDITOR in W-VIEW mode and add together $\frac{1}{2}$ and $\frac{1}{3}$

Write down the problem as $\frac{1}{2} + \frac{1}{3} =$ and write down the answer.

Do the same with $\frac{1}{5} + \frac{1}{7} =$ and $\frac{1}{4} + \frac{1}{6} =$

You should see a pattern (or rule) that will let you add two fractions together if both numerators are 1. Test your rule without the calculator by adding:

$\frac{1}{7} + \frac{1}{8}$ and $\frac{1}{10} + \frac{1}{3}$ Check your answers with the *WriteView* calculator.

We can generalise this rule for any two fractions whose numerators are 1:

$$\frac{1}{a} + \frac{1}{b} = \frac{1 \times b + 1 \times a}{a \times b}$$

Develop this rule for adding two fractions whose numerators are not 1.

$\frac{2}{5} + \frac{4}{9}$ and $\frac{3}{8} + \frac{2}{7}$ Check your answers with the *WriteView* calculator.

$$\frac{a}{b} + \frac{c}{d} = \frac{a \times d + c \times b}{b \times d}$$

A similar rule can be found for subtracting fractions.

Use the rule to find the answers to: $\frac{3}{6} + \frac{1}{4}$ and $\frac{1}{8} + \frac{5}{8}$

Check your answers using the *WriteView* calculator.

What do you notice? Use the *WriteView* calculator to investigate in turn:

$\frac{1}{2} =$, $\frac{8}{16} =$, $\frac{4}{8} =$, $\frac{10}{20} =$ Why should they all be the same ?

Are all the fractions $\frac{2}{6}$ $\frac{3}{9}$ $\frac{4}{12}$ $\frac{10}{30}$ the same ?

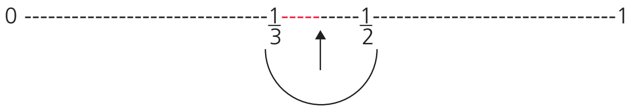
Which pairs of fractions $\frac{3}{4}$ $\frac{5}{20}$ $\frac{12}{16}$ $\frac{1}{2}$ $\frac{15}{30}$ $\frac{1}{4}$ are the same?

notes

HALFWAY BETWEEN

What fraction is halfway (midway) between $\frac{1}{3}$ and $\frac{1}{2}$?

Look at the number line below :



The halfway fraction can be found by finding the difference between the fractions $\frac{1}{2}$ and $\frac{1}{3}$ and adding half of that difference to the smallest fraction.

Using the *WriteView* calculator :

$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ and $\frac{1}{2}$ of $\frac{1}{6} = \frac{1}{12}$ and $\frac{1}{3} + \frac{1}{12} = \frac{15}{36} = \frac{5}{12}$

So :

$\frac{5}{12}$ is halfway between $\frac{1}{3}$ and $\frac{1}{2}$

What rule using the original fractions will give us the answer we need ?
Hint: look at 3 + 2 and twice (3 x 2)

What fraction is midway between $\frac{1}{4}$ and $\frac{1}{3}$?

Hint: look at 4 + 3 and twice (4 x 3). Check your answer using the *WriteView* calculator.

What fraction is halfway between $\frac{2}{3}$ and $\frac{3}{5}$?

The numerators are no longer both 1 so the rule will need to be altered to take this into account.

Hint: look at (2 x 5) + (3 x 3) and twice (3 x 5).
Check your answer on the *WriteView* calculator.

Write down the general rule for finding the fraction halfway between :

$\frac{a}{b}$ and $\frac{c}{d}$. Use this rule to find the fraction halfway between $\frac{4}{5}$ and $\frac{7}{8}$

Use the *WriteView* calculator to show that the answer given by the rule is correct.

notes

notes

ITERATION

Any equation can be solved by guesswork.
Iteration is the process of repeated, but increasingly better informed, guesswork.

The equation is rearranged to give an iteration formula and using this iteration formula repeatedly, a closer and closer value for the answer is achieved, stopping when the required degree of accuracy has been reached.

The process starts with an initial guess.

Find a positive value of $\mathbf{x}$ such that $\mathbf{x}^2 - 2\mathbf{x} - 1 = 0$.

Rearrange the formula: $\mathbf{x}^2 = 2\mathbf{x} + 1$ and then divide each term by $\mathbf{x}$ to give:

$\mathbf{x} = 2 + \frac{1}{\mathbf{x}}$ This is the basis of the iteration formula. $\mathbf{x}_{n+1} = 2 + \frac{1}{\mathbf{x}_n}$

which can be pictured as:

$\mathbf{x}_1 = 2 + \frac{1}{\mathbf{x}_0}$, $\mathbf{x}_2 = 2 + \frac{1}{\mathbf{x}_1}$, $\mathbf{x}_3 = 2 + \frac{1}{\mathbf{x}_2}$, ...

Starting from an initial guess $\mathbf{x}_0$, $\mathbf{x}_1$ will be the first solution, which will then be used to give $\mathbf{x}_2$ which will then be used for the next solution $\mathbf{x}_3$ and so on.

Let the initial guess for $\mathbf{x}_0$ be 2.

$\mathbf{x}_1 = 2 + \frac{1}{\mathbf{x}_0} = 2.5$

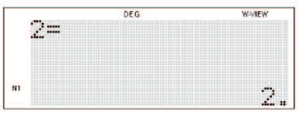
$\mathbf{x}_2 = 2 + \frac{1}{\mathbf{x}_1} = 2.4$

$\mathbf{x}_3 = 2 + \frac{1}{\mathbf{x}_2} = 2.4166667$

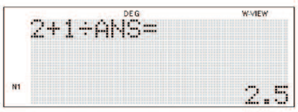
$\mathbf{x}_4 = 2 + \frac{1}{\mathbf{x}_3} = 2.413793103$

$\mathbf{x}_5 = 2 + \frac{1}{\mathbf{x}_4} = 2.414285714$

Iteration depends on using the last answer. The last answer feature of the calculator: ANS, makes iteration simple.



Stores the initial ANS as 2.



Sets up the iteration equation. Repeat pressings of = key carries out iteration.

A positive solution for $\mathbf{x}$ to 2 decimal places is 2.41.

The above equation has two solutions x = 2.41 and x = - 0.41.
Investigate the negative solution.
Use iteration to solve $\mathbf{x}^2 + \mathbf{x} - 4 = 0$.
What happens when you try and solve $\mathbf{x}^2 + \mathbf{x} + 4 = 0$? Explain why.

SHARP
www.sharp.co.uk/education

notes

ORDERING FRACTIONS

Draw a number line 0 to 2.
Estimate where on the line the following fractions should be placed:

$\frac{1}{2}$ $\frac{15}{8}$ $1\frac{2}{5}$ $\frac{3}{4}$ $\frac{13}{10}$ $\frac{3}{5}$

SETUP the EL-W531 EDITOR in W-VIEW mode and enter each fraction in turn: pressing = and then CHANGE to give the decimal equivalent.

Record these decimal equivalents against each fraction and write the decimals in order: smallest to largest.

Does this order match the order of your fractions on the number line ?

Notice how all your decimals have a fixed length – at most four digits long.

What happens with: $\frac{2}{3}$ $\frac{5}{6}$ when these are changed into decimals ?

Some fractions do not have neat decimals but continue with digits being repeated (or recurring).

Investigate $\frac{2}{9}$ $\frac{5}{9}$ $\frac{8}{9}$ as they are changed into recurring decimals.

You should see a pattern and be able to predict the decimal answers for:
$\frac{1}{9}$ $\frac{7}{9}$ $\frac{6}{9}$

Investigate the decimal equivalents of $\frac{2}{11}$ $\frac{3}{11}$ $\frac{4}{11}$

Again, there is a pattern. Not as simple as before. How is this last pattern connected to the number 9 ?

Predict the decimal equivalents of $\frac{5}{11}$ $\frac{6}{11}$ $\frac{7}{11}$

Explore the recurring decimal equivalents of

$\frac{4}{27}$ $\frac{5}{27}$ (three recurring digits) and $\frac{2}{13}$ $\frac{3}{13}$ (six recurring digits)

notes

PALINDROMES

A number that is the same backwards as forwards is a palindrome.

Examples are 1441, 3223, 535, 19891.

The same is true of words: noon, eye, racecar
and of sentences or phrases: never odd or even, dennis sinned, step on no pets are all palindromes.

Look at the four digit palindromes :

7667 4334 5225 1881

Divide each of them by 11.
What do you notice?

Look at the three digit palindromes

353 767 232 989

Divide each of them by 11.
What do you notice?

Using the idea of place value for a four digit palindrome **abba** the place values are given by:

$1000\mathbf{a} + 100\mathbf{b} + 10\mathbf{b} + \mathbf{a} = 1001\mathbf{a} + 110\mathbf{b}$

Can you show that all four digit palindromes are multiples of 11?

What general theory can you deduce about palindromes that have an even number of digits and an odd number of digits?

What can you say about the simplest palindromes (two digits) ie 22, 55, 44?

Can you find any three digit or five digit palindromes that are multiples of 11?

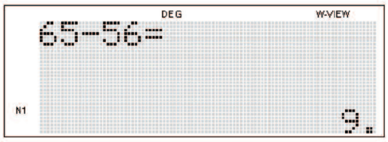
notes

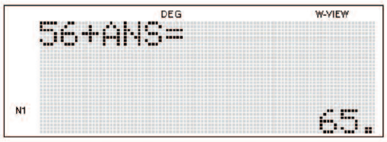
REVERSING THE ORDER

Using the calculator, the digits of any number can be reversed by always following a set sequence of calculations.

For example:
If we start with the number 56 then we want to finish with the number 65.

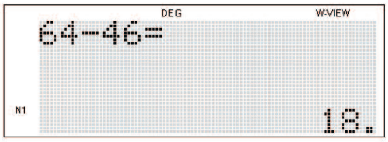
The difference between the largest: 65 and the smallest: 56 is 9.
If we **add** this difference to the smaller number we will get the reversed number.

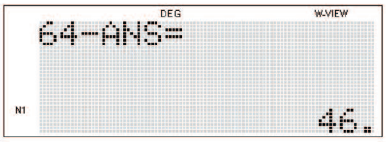




(Note: we can assign to key D1 the function ALPHA ANS so that D1 is all we need to press to recall the answer to the calculation)

Alternatively numbers can be reversed in this way.
Reverse the digits of 64 so that it becomes 46 .: **subtract** the difference from 64.





Investigate reversing the numbers 162 and 1541.
In general, one way to reverse the digits is to add or subtract the difference between the original number and its reversed form.

What do you notice about all the differences you have found? Use the up/down scroll keys to work back through your calculations.

For any two digit number **ab** the place value of **a** represents 10s and the place value of **b** represents units. When we reverse the digits to become **ba**, the place value of **b** represents 10s and the place value of **a** represents units.

The difference $(10\mathbf{a} + \mathbf{b}) - (10\mathbf{b} + \mathbf{a}) = 9\mathbf{a} - 9\mathbf{b} = 9(\mathbf{b} - \mathbf{a})$ is always a multiple of 9.

Investigate reversing three digit numbers **abc** and four digit numbers **abcd**
Is the difference always a multiple of 9?

notes

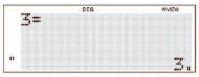
LAST ANSWER KEY – Generating Sequences

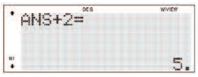
3 5 7 9 11

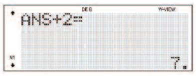
The sequence above has a pattern: it could be described as ‘add 2’

We can use the calculator ‘Last Answer’ facility: ANS, to explore sequences.

Starting from an initial input of 3 =, the rule ANS + 2 will generate the sequence.







The cursor keys ▲ and ▼ can be used to scroll through the sequence generated

Investigate these sequences and record the ANS rule that generates them:

5	9	13	17	…
2	4	8	16	…
24	19	14	9	…
243	81	27	9	…
1	2	5	14	…
1	2	5	26	…

Sometimes, more than one ANS rule can be found to generate a sequence.

Generate other sequences beginning:

1 2 …..

Fractional sequences can also be generated.

Starting from 1/12 =, the rule ANS + 1/12 will generate a sequence involving fractions, not all of which are in twelfths since the calculator will automatically simplify the fraction if possible.

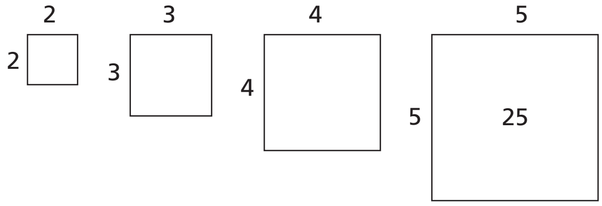
Investigate this rule and explain why the sequence generated shows halves, thirds, quarters and sixths.

Why, if we start with 1/7 =, and use the rule ANS + 1/7 do we only get fractions showing sevenths or whole numbers?

SHARP
www.sharp.co.uk/education

SQUARE ROOTS AND SURDS 1

Some numbers such as 4, 9, 16, 25, ….. are perfect squares

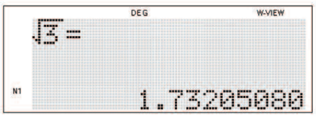


eg. the number 25 can be written as 5 x 5 : 5 is the exact square root of 25

For numbers other than perfect squares, no exact square root can be found. The square root of any positive whole number (integer) that is not a perfect square will be a decimal number that will never finish or be recurring.

$\sqrt{3} = 1.7320508075\ 6887729352\ 7446341505\ 8723669428\ 0525381038\ldots\ldots\ldots$

The calculator can only display ten digits so that $\sqrt{3}$ is shown as 1.73205080.



Square roots such as these are known as irrational numbers. The decimal can never be written as a fraction : a/b where a and b are integers.

Since the decimal never ends and no fraction exists which is exactly the same as the decimal, we leave the square root sign in place. Numbers which contain the square root are called surds and we use in these calculations.

$\sqrt{2}$ is a surd and an irrational number 1.414…………
$\sqrt{3}$ is a surd and an irrational number 1.372…………
$\sqrt{4}$ is a perfect square = 2

Sometimes a number can be written as two parts, one part of which is a perfect square : $32 = 16 \times 2$ so that $\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$.
$\sqrt{32}$ is still an irrational number because it is not a perfect square.

We can work with surds in a similar way to ordinary numbers :

$\sqrt{3} \times \sqrt{2} = \sqrt{(3 \times 2)} = \sqrt{6}$, $\quad \sqrt{10} \div \sqrt{2} = \sqrt{(10 \div 2)} = \sqrt{5}$, $\quad 4\sqrt{3} \times 3\sqrt{2} = 12\sqrt{6}$

Sometimes an irrational number times an irrational number will produce a rational (real number) answer :

$\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$. Investigate $\sqrt{6} \times \sqrt{3} \times \sqrt{2}$ and explain the real number answer.

Explain the answer to $\sqrt{2} \times \sqrt{2}$. What is the answer to $\sqrt{a} \times \sqrt{a}$?

notes

notes

SQUARE ROOTS AND SURDS 2

All irrational numbers, written by the calculator using surds, can be changed to their decimal equivalents – the first ten digits of which can be shown on the calculator display.

In any fraction in which the denominator could contain a surd, the logic of the calculator will always show the denominator as an integer.

Investigate the way the calculator displays: $\frac{1}{\sqrt{3}}$ =, $\frac{1}{\sqrt{2}}$

Explain why $\frac{1}{\sqrt{2}}$ is the same as $\frac{\sqrt{2}}{2}$

(Hint : Use the information of last line **SQUARE ROOTS and SURDS 1**)

Using the rule that we had for adding fractions and rewriting $\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{3}}$

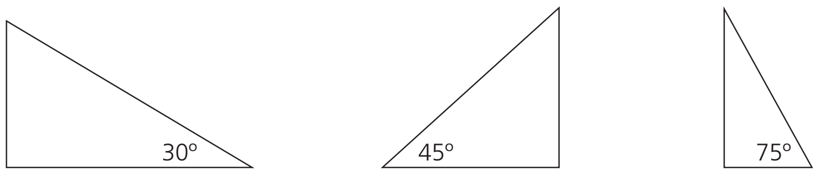
in the way that the calculator does explain why :

$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{6}}$ (using the rule for adding fractions)

should be the same as the calculator answer of $\frac{2\sqrt{3} + 3\sqrt{2}}{6}$

Some trigonometric ratios of certain angles are irrational numbers and are written using surds.

Investigate cos 30°, tan 45°, sin 75° and cos 75°



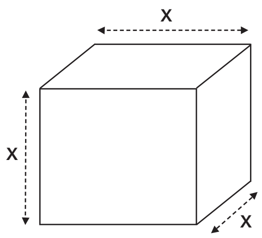
If, within a calculation, the result involves the square root of a negative number the calculator will give an -ERROR 02- display.

For any display containing surds the approximate decimal equivalent answer can easily be found using the CHANGE key.

notes

TRIAL AND IMPROVEMENT

'Trial and Improvement' is the name given to method of finding the answer to a problem by making an initial first guess and then improving on it by looking at the answer obtained.



For example:

Find the size of a cube-shaped box that will hold $100cm^3$ of liquid.

Since the box is a cube, each side has the same length: **x** cm. The volume of the box 100 cm^3 is given by $\mathbf{x}^3$

Guess at the length of **x** as 5 cm.
The volume of the box would be 5 x 5 x 5 = 125 cm^3 . Too big.
Make x smaller , say 4 cm. Volume is now 4 x 4 x 4 = 64 cm^3. Too small.
Try again: x = 4.5 cm. Volume is now (4.5)3 = 91.125 cm^3. Getting better.

Keeping your guesses to one decimal place, use you calculator to get the best value for x that you can.

Change your guesses now to two decimal places to find a better value.

Change you guesses now to three decimal places.

The answer you choose to use will always depend on the degree of accuracy that you require.

$4.640^3 = 99.897344$	The answer for x lies between 4.641 and 4.642.
$4.641^3 = 99.96194672$	The mid point value x = 4.6415 gives a volume of
$4{,}642^3 = 100.0265773$	99.99445852, too small to hold 100 cm3 of liquid.
$4.643^3 = 100.0912357$	So to three decimal places the value of x is 2.642

The 'best' answer from the calculator, which shows answers to ten figures in Normal mode is given by $\sqrt[3]{100}$ = 4.641588834.

Use trial and improvement to find a positive value of **x** such that $\mathbf{x}^2 - 2\mathbf{x} - 1 = 0$
Give your answer for **x** to two decimal places.

Another technique for solving equations is called Iteration, this uses the idea of trial and improvement but in a more controlled way.

notes

TRIPLES

Using the co-ordinate (Cartesian and Polar) capabilities of the calculator:

What is the hypotenuse of a right-angled triangle whose shorter sides are

1 9 cm and 12 cm?

2 9 cm and 40 cm?

3 13 cm and 84 cm?

In the above examples, all three sides of the right-angled triangle are integers. These are examples of Pythagorean triples. A Pythagorean triple can be found by squaring any odd integer and splitting the answer into two consecutive integers.

$9^2 = 81$ $81 = 40 + 41$ The 9, 40 and 41 form a Pythagorean triple.

$13^2 = 169$ $169 = 84 + 85$ The 13, 84 and 85 form a Pythagorean triple.

Find Pythagorean triples starting with :

17^2 7^2 and 11^2

Confirm your answers using the two smaller integers of the triple and the coordinate features of the calculator.

Alternatively, starting with the square of any even integer: divide the square by two and split the answer into two 'close' integers.

$6^2 = 36$ $36 \div 2 = 18$ $18 = 10 + 8$ 6, 8 and 10 is the triple

Find Pythagorean triples starting with

14^2 12^2 and 8^2

Confirm your answers by using the 'coordinate technique'.